Antiderivatives.

$F(x)$ is an antiderivative of $f(x)$ if

$$F'(x) = f(x).$$

$f(x)$ is the derivative of $F(x)$

$F(x)$ is an antiderivative of $f(x)$

$F(x) + C$ is also an antiderivative of $f(x)$

... and we write

$$F(x) + C = \int f(x)\,dx$$

the constant of integration

read "integral of $f(x)$"

"indefinite integral of $f(x)$"

"antider . . . . "

$$\int c\,f(x)\,dx = c\int f(x)\,dx$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Power rule for antiderivatives

Differentiate x^n

(1) multiply by exponent

(2) subtract 1 from exponent

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Antiderivative of x^n

(1) add 1 to exponent

(2) divide by new exponent

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int x^3\,dx = \frac{x^4}{4} + C$$

$$\frac{d}{dx}\left(\frac{x^4}{4} + C\right) = \frac{1}{4}\frac{d}{dx}\left(x^4\right) + \frac{d}{dx}(C)$$

$$= \frac{1}{4}\left(4x^3\right) + 0$$

$$= x^3$$

① $\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{d}{dx}\left((1+x^2)^{1/2}\right) =$

$\frac{1}{2}(1+x^2)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{1+x^2}}$

$\int \frac{x}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2} + C$

~~Harvard University Girl~~

$F(x)$ is an antiderivative of $f(x)$ if

$F'(x) = f(x)$.

$f(x)$ is the derivative of $F(x)$.

$F(x)$ is an antiderivative of $f(x)$.

So is
$F(x)+1$
$F(x)-20$
$F(x)+1000000\pi$
⋮

and we write

$$F(x) + C = \int f(x)\,dx$$

C: Constant of integration

read "the integral of $f(x)$"
"the antiderivative of $f(x)$"
"the indefinite integral of $f(x)$"

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

$$\int 3x^2\,dx = x^3 + C$$

$$3\int x^2\,dx = x^3 + C$$

$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\int 4x^2\,dx = 4\left(\int x^2\,dx\right) = 4\left(\frac{x^3}{3} + C\right)$$

$$= \frac{4}{3}x^3 + C$$

$$\int c f(x)\,dx = c\int f(x)\,dx$$

$$\frac{d}{dx}\left(\int 3x^2\,dx\right) = 3x^2$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

moral: Integration is difficult

No specific rule for

- multiplication (but $\exists$ a process)
- division
- composition (yet)

Power rule for integration

Derivative of x^n

1. mult by old exponent
2. subtract 1 from exponent

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Integral of x^n

1. add 1 to the exponent
2. divide by new exponent

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\iiiint x^2\,dx$$

6.2/1

$$\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{d}{dx}\left(1+x^2\right)^{1/2}$$

$$= \frac{1}{2}\left(1+x^2\right)^{-1/2}\left(2x\right)$$

$$= \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2} + C$$

6.2/41 a. $\frac{dy}{dx} = \sqrt[3]{x}$; $y(1) = 2$

$y = \sim$

$\frac{dy}{dx} = \frac{d}{dx}(y)$

$\frac{d}{dx}(\sim)$

$$\frac{dy}{dx} = x^{1/3}$$

$$y = \frac{x^{4/3}}{4/3} + C$$

$$y = \frac{3x^{4/3}}{4} + C$$

$$(y=2) = \frac{3(x=1)^{4/3}}{4} + C$$

$$2 = \frac{3}{4} + C$$

$$2 - \frac{3}{4} = \frac{5}{4} = C$$

$$y = \frac{3x^{4/3}}{4} + \frac{5}{4}$$